Wednesday, April 19, 2017
7:06 PM

To maximally extend the Kerr geometry, we could adopt Kruskal-type coordinates, but let's go ahead and set up a more powerful tool.

Conformal (or Penrose) diagrams:

$\mathbb{M}^{4}$ $\quad ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 \qquad t \in (-\infty, \infty),\ r \in [0, \infty)$

Let's grab infinity and snuggle up to it w/

$$\left.\begin{aligned} T &= \tan^{-1}(t+r) + \tan^{-1}(t-r) \\ R &= \tan^{-1}(t+r) - \tan^{-1}(t-r) \end{aligned}\right. \Rightarrow \left.\begin{aligned} &0 \leq R < \pi \\ &|T| < \pi - R \end{aligned}\right\} \text{ both have finite ranges}$$

Then: $\displaystyle ds^2 = \frac{1}{(\cos T + \cos R)^2}\left(-dT^2 + \underbrace{dR^2 + \sin^2 R\, d\Omega^2}\right)$

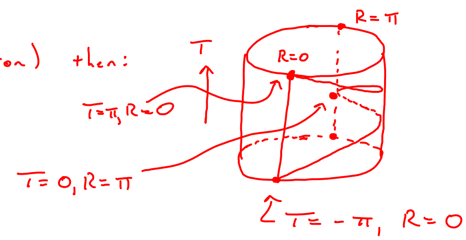
Note R is acting like an angle

or

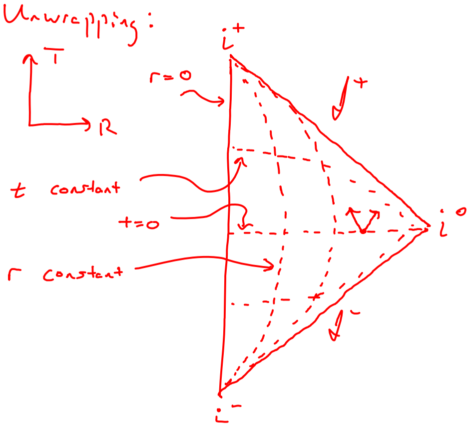
$\displaystyle ds^2 = \frac{1}{\omega^2(T,R)}\, d\tilde{s}^2$

conformally related geometry (preserves angles, in particular light-cones!!)

To visualize it, suppress θ + ϕ (purely radial motion) then:



Unwrapping:



i^+ = timelike future ∞ $\quad\Big\}$ All timelike geodesics ($m>0$)
i^- = timelike past ∞ $\quad$ begin and end here

i^0 = spacelike ∞ $\quad$ All spacelike geodesics begin and end here

$\mathscr{J}^+$ = future null ∞ $\quad\Big\}$ All lightlike geodesics ($m=0$)
$\mathscr{J}^-$ = past null ∞ $\quad$ begin and end here

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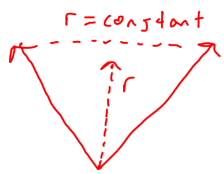
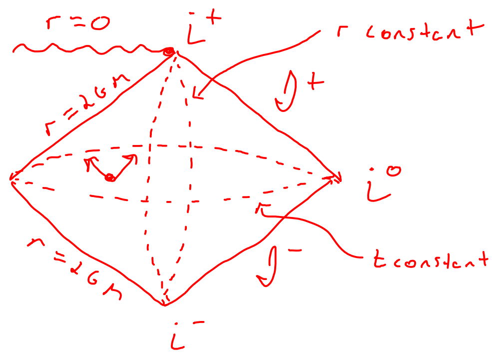
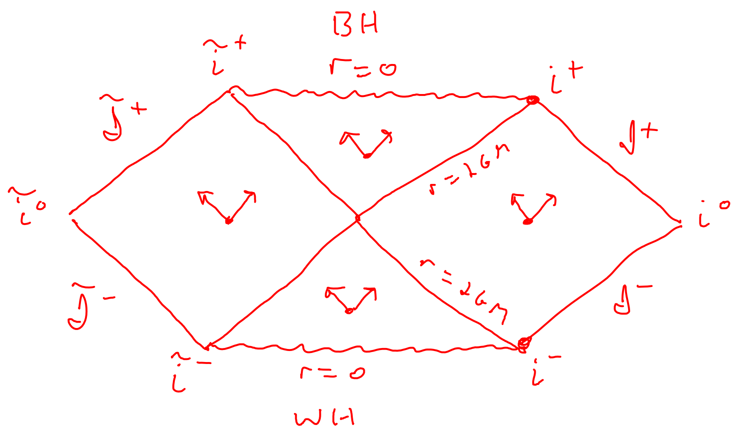
For Schwarzschild:

The singularity in this case is in a region where r is timelike, i.e. $(-)dr^2$

In terms of light cones:







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For Kerr:

The global geometry of the maximally extended Kerr solution is more complicated for 2 reasons:

a) It looks different for different choices of how a^2 compares to G^2M^2

b) Perhaps even stranger is that looks different for different values of θ!

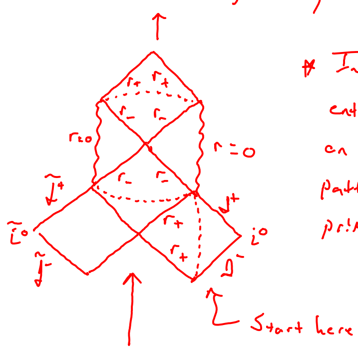
- We know $a=0$ reduces to Schwarzchild

- $a^2 < G^2M^2$ Recall: $\rho^2 = r^2 + a^2\cos^2\theta$

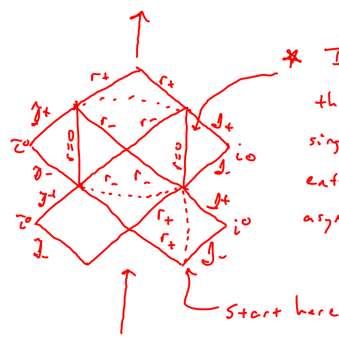
$\theta = \frac{\pi}{2} \Rightarrow r=0$ true singularity

$\theta \neq \frac{\pi}{2} \Rightarrow r=0$ is nonsingular

* Curves of constant r are dashed lines.
* Light cones at 45° everywhere.
* Singularities in this case are spacelike since $(+)dr^2$.

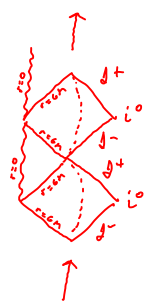


* In both cases the entire solution is an ∞-long repeating pattern of these primitive regions!



* If you pass through the ring singularity, you enter a different asymptotic spacetime!

- $a^2 = G^2M^2$



Notice that you can go inside and exit the horizon, but then you enter a different asymptotic spacetime. You saw this on your HW!

- $a^2 > G^2M^2$ For reasons we will learn about next time, there is no reason to consider this case.